Statistical Analysis 8: Two-way analysis of variance (ANOVA)

Research question type: Explaining a continuous variable with 2 categorical variables

What kind of variables? Continuous (**scale/interval/ratio**) and 2 independent categorical variables (**factors**)

Common Applications: Comparing means of a single variable at different **levels** of two conditions (**factors**) in scientific experiments.

Example:

The effective life (in hours) of batteries is compared by material type (1, 2 or 3) and operating temperature: Low (-10˚C), Medium (20˚C) or High (45˚C). Twelve batteries are randomly selected from each material type and are then randomly allocated to each temperature level. The resulting life of all 36 batteries is shown below:

Table 1: Life (in hours) of batteries by material type and temperature

		Temperature (˚C)		
		Low (-10˚C)	Medium (20˚C)	High (45˚C)
Material type	1	130, 155, 74, 180	34, 40, 80, 75	20, 70, 82, 58
	2	150, 188, 159, 126	136, 122, 106, 115	25, 70, 58, 45
	3	138, 110, 168, 160	174, 120, 150, 139	96, 104, 82, 60

Source: Montgomery (2001)

Research question: Is there difference in mean life of the batteries for differing material type and operating temperature levels?

In analysis of variance we compare the variability **between** the groups (how far apart are the means?) to the variability **within** the groups (how much natural variation is there in our measurements?). This is why it is called analysis of variance, abbreviated to **ANOVA**.

This example has two **factors** (material type and temperature), each with 3 **levels**.

Hypotheses:

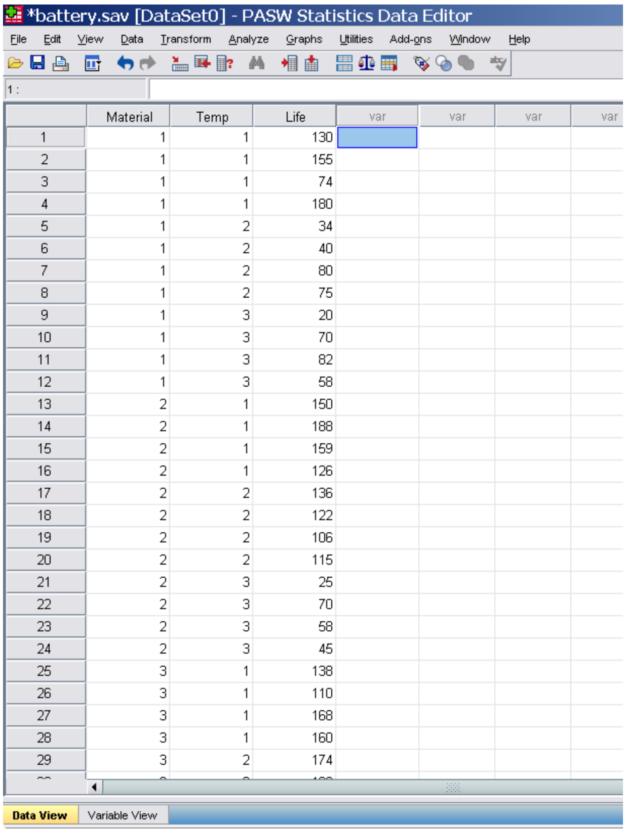
The 'null hypothesis' might be:

H_0: There is **no difference in mean** battery life for different combinations of material type and temperature level

And an 'alternative hypothesis' might be:

H_1: There **is a difference in mean** battery life for different combinations of material type and temperature level

If the alternative hypothesis is accepted, further analysis is performed to explore where the individual differences are.



Steps in SPSS (PASW):

Data need to be arranged in SPSS in a particular way to perform a two-way ANOVA. The dependent variable (battery life) values need to be in one column, and each factor needs a column containing a code to represent the different levels.

In this example *Material* has codes 1 to 3 for material type in the first column and *Temp* has codes 1 for Low, 2 for Medium and 3 for High operating temperatures.

The battery life (*Life*) is entered in the third column – see screen to the left.

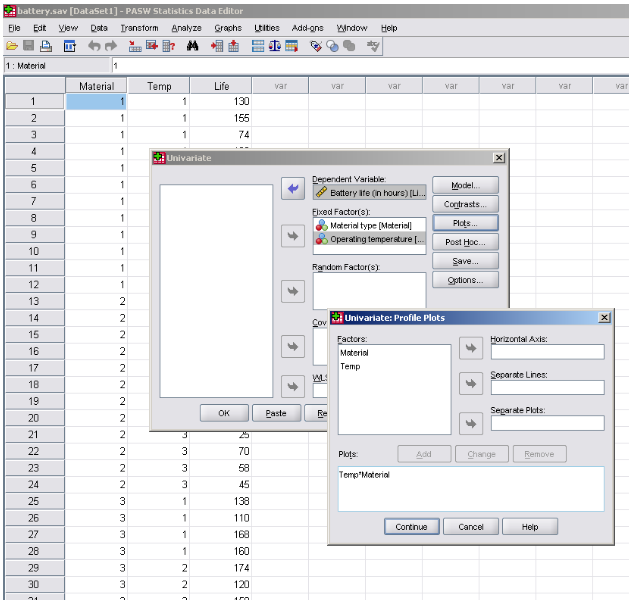
Note carefully how the data are entered.

The raw data file for this example is available on W:\EC\STUDENT\ MATHS SUPPORT CENTRE STATS WORKSHEETS\battery.sav

Then choose:
Analyze > General Linear Model > Two-Way ANOVA…

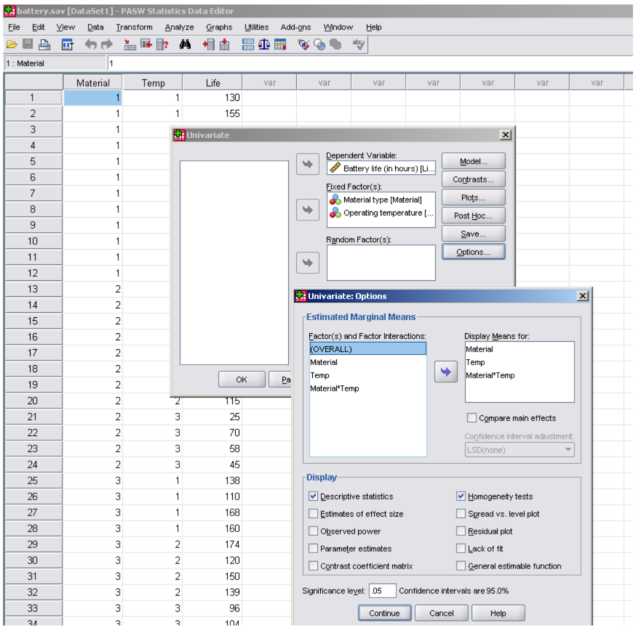
- Transfer the outcome variable (*Life* in this example) into the Dependent Variable box, and the factor variables (*Material* and *Temp* in this case) as the Fixed Factor(s)
- Click on **Model…** and select Full factorial to get the 'main effects' from each of the two factors and the 'interaction effect' of the two factors. [It is possible to build a Custom model, if you prefer]
- Continue

- Click on **Plots…**, and choose Temp for Horizontal Axis and Material in Separate Lines (see right)
- Click Add and Continue



- Click on **Post Hoc…** and select *Material* and *Temp*
- Check Tukey (or post hoc test of choice)
- Continue

- Click on **Options…** and choose to Display Means for *Material*, *Temp* and *Material**Temp*
- Check Descriptive statistics and Homogeneity tests (see right)
- Continue and OK



In the SPSS output there is a table showing the **descriptive statistics** for the main variable (battery life) at each of the levels for each factor (9 in this example), plus Totals.

Check the result of **Levene**'s test for a p-value (Sig.) > 0.05, so that similar variances for each group of measurements can be assumed (otherwise the ANOVA is probably invalid).

In the example, p = 0.529, so the two-way ANOVA can proceed.

The **Tests of Between Subjects Effects** table gives the results of the ANOVA. Table 2 below shows the output for the battery example with the important numbers emboldened.

Table 2: Tests of Between-Subjects Effects

Dependent Variable: Battery life (in hours)

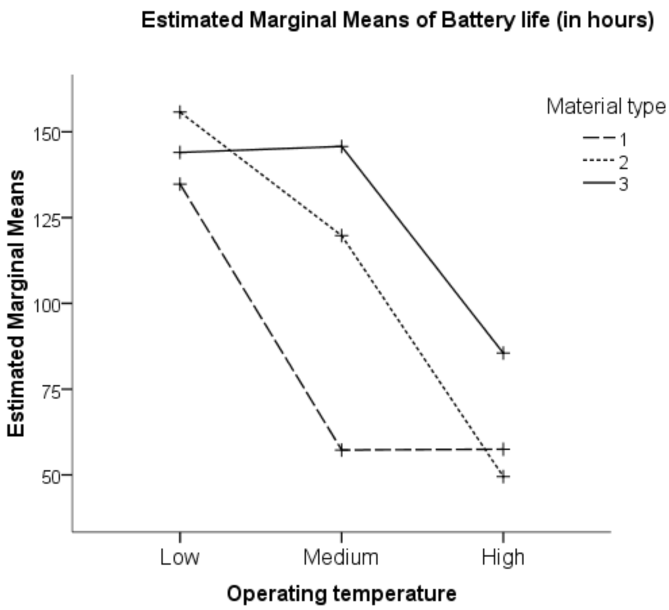
Source	Type III Sum of Squares	df	Mean Square	F	**Sig.**
Corrected Model	59416.222[a]	8	7427.028	11.000	.000
Intercept	400900.028	1	400900.028	593.739	.000
Material	10683.722	2	5341.861	**7.911**	**.002**
Temp	39118.722	2	19559.361	**28.968**	**.000**
Material * Temp	9613.778	4	2403.444	**3.560**	**.019**
Error	18230.750	27	675.213		
Total	478547.000	36			
Corrected Total	77646.972	35			

***p*-values**

Results:

From the Descriptive Statistics table, it can be seen that, overall, battery life decreases with higher operating temperature, although battery life remains high for material 3 at medium temperature. This pattern is more obvious when looking at the **plot** – see right.

Since the lines representing the three materials in the plot are not parallel, this implies there is an interaction effect between material and operating temperature. [The lines would be approximately parallel if there were no interaction.] So, how battery life changes with temperature depends on the material, and vice versa.



The ANOVA table gives F statistics = 7.91, p=0.002; 28.97, p<0.001 and 3.56, p=0.019, for material, operating temperature and material*temperature, respectively [**NEVER write p = 0.000**]. So**, both** material and temperature are needed, as well as their **interaction**, to explain battery life.

The nature of these differences can be explored further by looking at the SPSS output from the '**post hoc**' tests. These suggest that mean battery life overall is statistically significantly longer for material 3 than 1 (p=0.001), and at lower compared to higher temperature levels. However, at low operating temperatures material 2 appeared to give a longer life than for materials 2 and 3, but lasted least at high temperatures.

Conclusion:
From the results it can be said that there is strong evidence that the mean battery life varies with material used and operating temperature (p=0.002 and p<0.001). The presence of interaction between material and temperature means that the way battery life changes for different materials depends on the temperature. Similarly, the way battery life changes for different temperatures depends on material. Overall, material 3 performs best.

The three tables of estimated marginal means give details of mean battery lives by factor, plus 95% CIs, giving more detail concerning the accuracy of these battery life estimates from the sample to the overall population. In this example, the CIs are all fairly 'wide', so results should be used with caution.

Validity of two-way ANOVA:

ANOVA is based on two assumptions:

- the observations are random samples from normal distributions
- the populations have the same variance [variance = (standard deviation)^2]
- observations are independent of each other

So, before carrying out any tests the data must be examined in more detail to determine whether these assumptions are satisfied. See one-way ANOVA sheet for more information relating to this aspect.

Comments:

- Multiple t-tests should not be performed
- It is possible to perform two-way ANOVA with different sample sizes per group. Select Type IV Sum of squares in the Univariate: Model dialog box.

Montgomery DC (2001) *Design and Analysis of Experiments* (5th Ed.) John Wiley & Sons Inc., New York